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REFRACTIONS AND REFLECTIONS

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Practical Astigmatism Analysis for Refractive Outcomes in Cataract and Refractive Surgery

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Abstract. The fundamental concepts underpinning the vectorial analysis of astigmatism are straightforward and intuitive, easily understood by employing a simple golf-putting analogy. The Alpins methodology utilizes three principal vectors and the various ratios between them to provide an aggregate analysis for astigmatic change with parallel indices for spherical correction. A comparative analysis employing both arithmetic and vectorial means together with necessary nomogram adjustments for refining both spherical and astigmatic treatments can also be derived. These advanced techniques, together with their suitability for statistical analysis, comprehensively address the outcome analysis requirements of the entire cornea and the eye's refractive correction, for the purpose of examining success in cataract and refractive surgery. (**Surv Ophthalmol 49**:109–122, 2004. © 2004 Elsevier Inc. All rights reserved.)

Key words. astigmatic vectorial difference • correction index (CI) • difference vector (DV) • flattening effect (FE) • index of success (IOS) • target induced astigmatism vector (TIA) • torque effect

Basic Principles of Vector Analysis

Vectors are mathematical expressions that combine values for magnitude and direction. A given vector has specified values for each of these parameters, which are the classical descriptors of the physics of motion. For example, a given force has a specified magnitude and direction. However, vectors can also be applied wherever the combination of magnitude and direction is required in a single mathematical expression. Astigmatism, with cylinder power and axis (refractive) or magnitude and meridian (corneal), fits such a description.

Manipulation of vectors follows certain rules and can yield resultant vectors from combinations of others. For example, several forces acting on a body can be resolved to a single summated vector if the magnitude and direction of each is known. Similarly, the combination of preoperative astigmatism and surgical effect on astigmatism can yield postoperative astigmatism. More usefully, in the clinical setting, if the pre- and postoperative astigmatism is known, the effect that surgery has had on astigmatism can be calculated.

Methods of Vector Analysis

The basic methods for analyzing vectors are graphic and trigonometric.

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GRAPHIC ANALYSIS

Graphic analysis consists of drawing lines in the appropriate direction and of a length governed by a scale of length to the unit of magnitude of interest, which for astigmatism is the dioptric power. They can be added, as in Fig. 1, where vector ***b*** is added to the head of vector ***a***. The resultant vector ***c*** can then be determined for direction and magnitude. Vectors may also be resolved into component vectors, as in Fig. 2, where vector ***c*** can be seen to be composed of vectors ***a*** and ***b*** with tails at right angles to each other placed, for example, on the horizontal *x* and vertical *y* axes. A vector may be changed by altering either its magnitude (or length, in graphic terms) or its direction. In Fig. 3 (left and right), the transposition of the vector from one point to another does not affect its identity; ***d*** is the same vector in both displays. Lastly, if the vector required to convert one vector to another is to be established, that vector would connect the head of the first vector to the head of the second, and it could then be drawn and measured (Fig. 4 [left and right]). This forms the basis for calculation of the sum of two obliquely crossed cylinders. These techniques can be employed where multiple additions are required for the analysis of aggregate astigmatism data and will be described in more detail later.

TRIGONOMETRIC ANALYSIS

The alternative method for manipulating vectors is trigonometric. Trigonometric mathematics rests on the fixed proportions of a right triangle (Fig. 5). The ratio of the length of the side opposite an angle α to the hypotenuse is fixed for any given value of α and is called the sine of α. The ratio of the adjacent side to the hypotenuse is also fixed and is the cosine of α, and the ratio of the opposite to the adjacent is the tangent of α (Fig. 5). All angles and sides are fixed and calculable if one angle and one side length are known, as long as the triangle is right-angled. By constructing such triangles about vectors of interest, using known values, we can calculate vector values without recourse to the tedious graphical method.

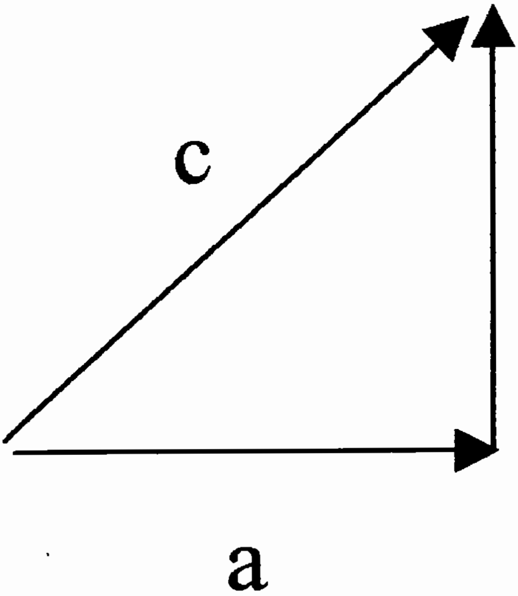


Fig. 1. Graphic analysis trigonometry: two components and their resultant vector.

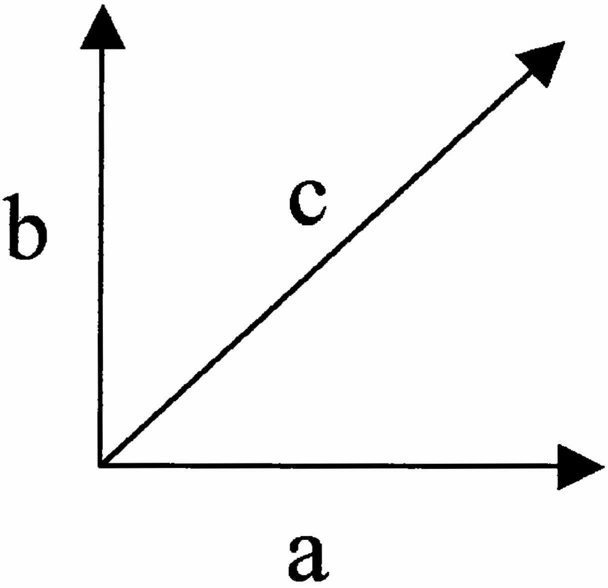


Fig. 2. Graphic analysis trigonometry: resolution of a vector into its two orthogonal components.

Using vector analysis, we can combine the numerical descriptors of refractive astigmatism (cylinder power and axis) in a unified mathematical expression (a vector with magnitude and direction), allowing us to calculate astigmatic change in terms of power/magnitude and axis/direction, where these two values vary in an interdependent way. This allows us to look at each of the two components independently.

Even though astigmatism and vectors share the same units of measurement, diopters and degrees, it is important to understand that they possess fundamentally different properties: astigmatism is a static entity that is measurable on a toroidal surface and a vector is a dynamic entity whose two components of magnitude and axis cannot be measured—they can only be calculated. Simple arithmetical calculations between astigmatism and vectors are not valid.

CALCULATION OF THE SURGICALLY INDUCED ASTIGMATISM VECTOR

The summation of two obliquely crossed planocylindrical lenses provides the fundamental formula

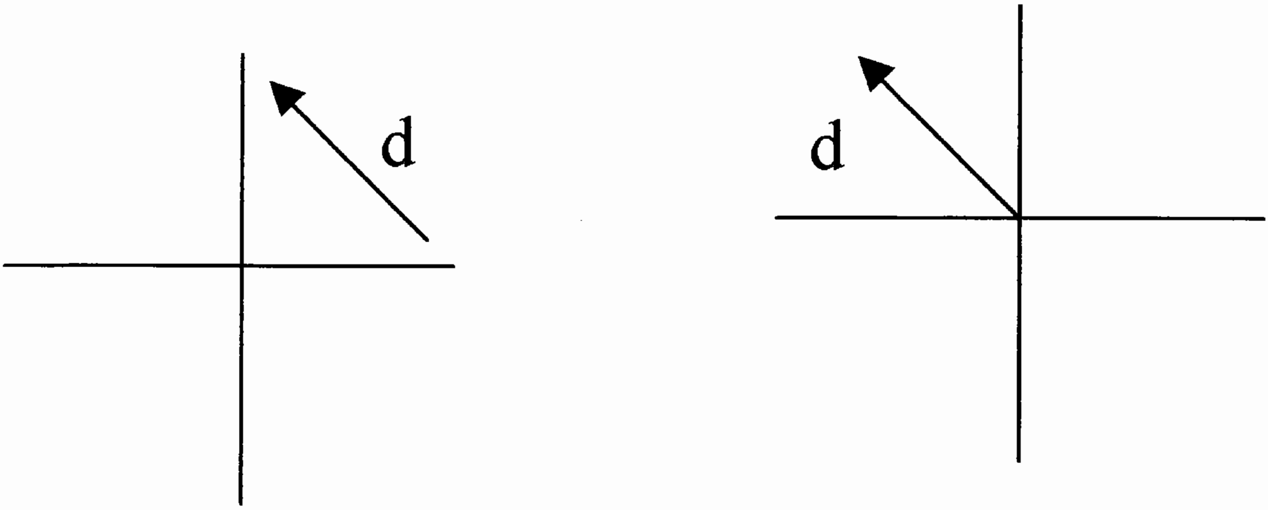


Fig. 3. A and B: Graphic analysis trigonometry: vector transposition without a change of identity.

for calculating the surgically induced astigmatism vector (SIA).[41] Researchers based in the optical sciences, including Gartner[13] and Naylor,[38] employed graphical analysis to confirm the veracity of the formula to calculate the magnitude and axis of the astigmatic change. Jaffe and Clayman, in their seminal paper, described the use of rectangular and polar coordinates to determine by trigonometric analysis the formula for calculating the magnitude of the SIA and its axis, with the known values for pre-operative and postoperative corneal astigmatism.[22]

THE DOUBLE ANGLE VECTOR DIAGRAM

The examination of any change in astigmatism may not be clearly apparent on a polar diagram. On initial examination of a straightforward example, such as, from a to b (Fig. 6A) this may appear to be a big change, but when a vectorial comparison is made on a vector diagram by doubling astigmatism axes, (Fig. 6B), the actual change is relatively small, shown by the dashed line from a to b on a head-to-head evaluation. This is a measure of the astigmatic change induced by surgery (SIA), but it is also a useful technique to compare two differing spherocylindrical lenses, such as manifest refraction versus wavefront refraction (predicted phoroptor refraction) values, or two differing techniques for corneal astigmatism measurement. This comparison process is termed astigmatic vectorial difference. This example demonstrates well that examining astigmatism axis or astigmatism magnitude separately on a polar diagram to quantify change, are inconclusive and can be misleading. To avoid such confusion a double-angle vector diagram (DAVD) should be created. This is a mathematical and graphical device to overcome the possible errors induced by vector calculation using the conventional 180° or polar representation of cylinder axis. All axis values are doubled before calculations are made on the DAVD, and then halved after calculations are completed, to obtain the clinically relevant, conventional polar axis. One expression of the trend in astigmatism values in a population can be obtained by head-to-tail summation, as shown in Fig. 6C.

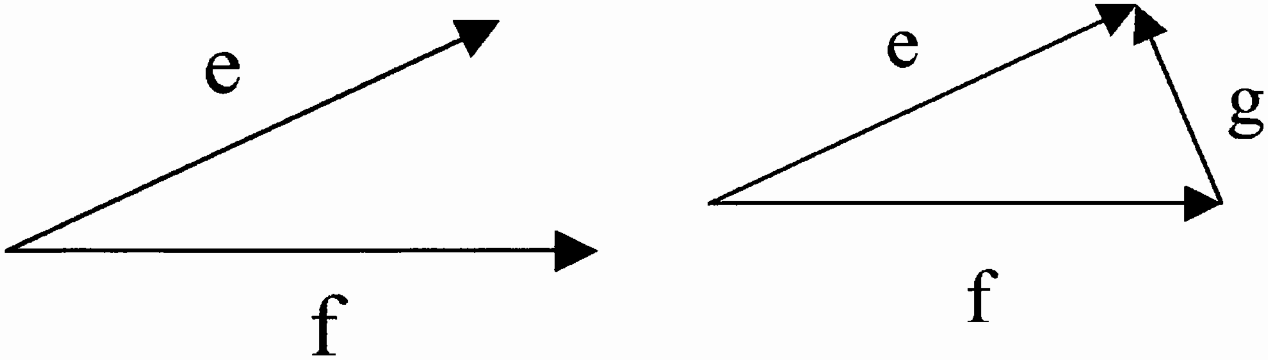


Fig. 4. A and B: Graphic analysis trigonometry: addition of obliquely crossed cylinder: g must be added to f for it to become e. When comparing cylinder values, g is the astigmatic vectorial difference between e and f $(e - f)$.

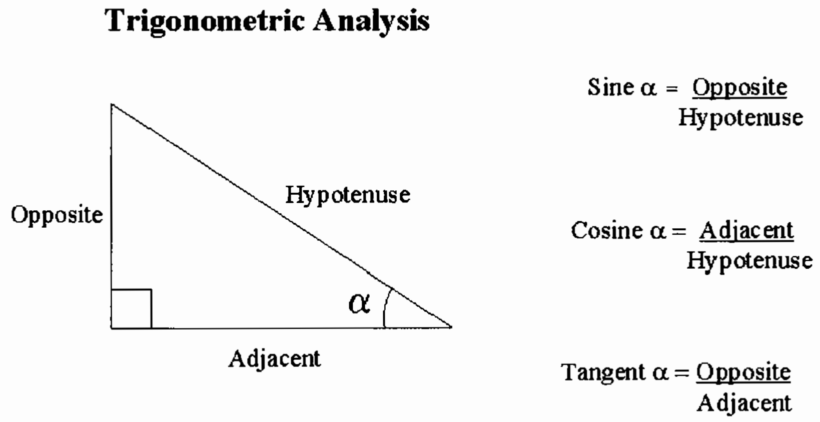


Fig. 5. Trigonometric analysis: solution of the right triangle with one known angle and side.

The primary data for trigonometric vector analysis are rectangular and polar coordinates. These can be calculated and formulas to establish the vectors they represent, can be applied as set out below.

Rectangular Coordinate (Double-Angle Vector Diagram) Formulas

Formula for Calculation of SIA

The well-recognized formula described by Jaffe[22] for vector analysis is:

$$K_{13} = (K_1{}^2 + K_3{}^2 - 2K_1K_3\text{cosine}2[\theta_1 - \theta_3])^{1/2}$$

where

K_{13} = surgically induced astigmatism vector magnitude

K_1 = preoperative astigmatism magnitude

K_3 = postoperative astigmatism magnitude

θ_1 = preoperative astigmatism steepest meridian

θ_3 = postoperative astigmatism steepest meridian

where K_1 and K_3 are the dioptric difference between the steepest and flattest curvatures of the cornea or maximum and minimum powers of the correcting spherocylindrical lens.

This formula precisely analyzes the magnitude and direction of changes induced by individual incisions or ablations, and is therefore extremely important for analyzing astigmatic change in whichever orientation it occurs. The SIA formula assigns absolute values to each change induced, regardless of the axis, thus producing a value for the total astigmatic change in the eye. Because the SIA formula derives a positive number and a specific axis, a useful arithmetic or vector summated average of these magnitude values can be calculated. Negative vector values are dealt with by vectorially reversing the vector to the opposite quadrant (180° change on a double-angle vector diagram, a 90° change on a polar diagram) to obtain the true orientation of the vector. However, calculating an arithmetic average of multiple values of this total induced astigmatism requires that the orientation of individual axes be ignored. An advantage of this formula is that it treats the oblique and polar axes equally. It represents the induced astigmatism in a consistent fashion, whatever the meridian of preoperative astigmatism (Figs. 7A and 7B).

One limitation of this method is that whereas the formula accurately gauges total astigmatic change, it does not enable the surgeon to quantify the final postoperative status of astigmatism using the formula alone without a preoperative astigmatism value. As a consequence, a second limitation is that the formula alone provides no indication of how successful the surgical outcome was, what errors occurred, and what adjustments might be made to future surgeries to achieve improved results. Furthermore, a formula based on the law of cosines sometimes makes difficult the identification of the quadrant in which the axis of the SIA belongs.[9,23,45]

Where a change in the astigmatic state of the eye is intended, it is the various relationships between the SIA and the astigmatic treatment of choice, quantified by the target induced astigmatism vector (TIA), that tells us whether the treatment was on-axis or off-axis, and whether too much or too little treatment was applied.[9]

Alpins Method

Formulas published by Alpins[6,9] encompass all the requirements of the polar analysis formulas described below, but belong to the rectangular coordinate group. Three fundamental vectors are employed (Fig. 8), whose functionality is readily understood using a golfing analogy.

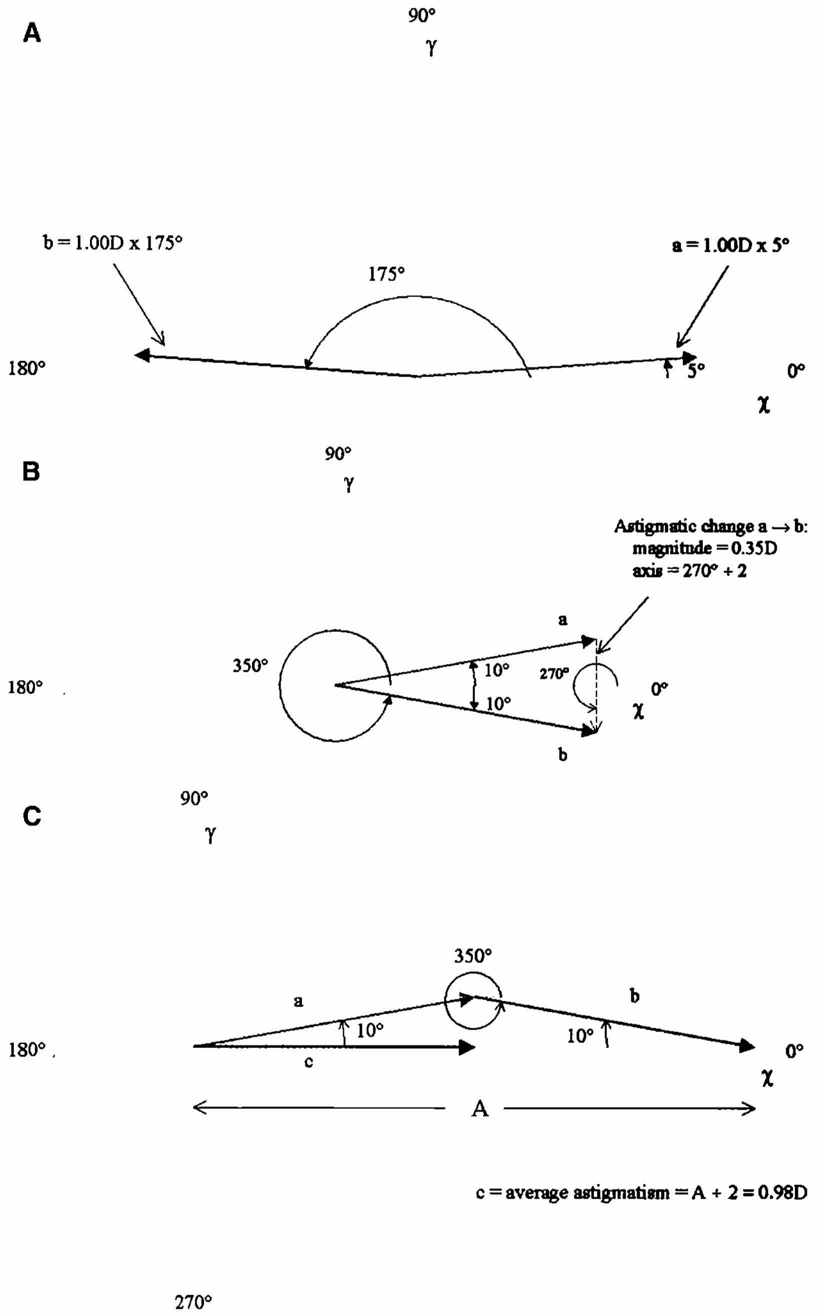


Fig. 6. *A:* Polar astigmatism diagram: examination of an apparent large change in astigmatism values can be misleading when displayed on a polar astigmatism diagram alone. *B:* Double-angle vector diagram: calculation of change of astigmatism by vector analysis (tail-to-tail) reveals only a relatively small surgical change has occurred and is quantified by the SIA. *C:* Double-angle vector diagram: calculation of average astigmatism value by vector summation (head-to-tail) to show the overall trend.

Any new concept that expands the existing understanding of a subject requires many qualities to gain general acceptance. The underlying concept should be simple to understand. Any inadequacy in past understanding of a subject should readily become apparent. New information provided by the introduction of a concept should be useful and easy to apply. The acceptance of a new analytical technique will be enhanced if its application assists in the decision-making process for planning as well as quantifying the effect and success of surgical outcomes.

Hence the golfing analogy. Putting a golf ball into the cup on a flat green is a simple process to understand, but not always easy to accomplish. Golf-putting shares common concepts with the analysis of astigmatism. A golf putt is similar to a vector that induces

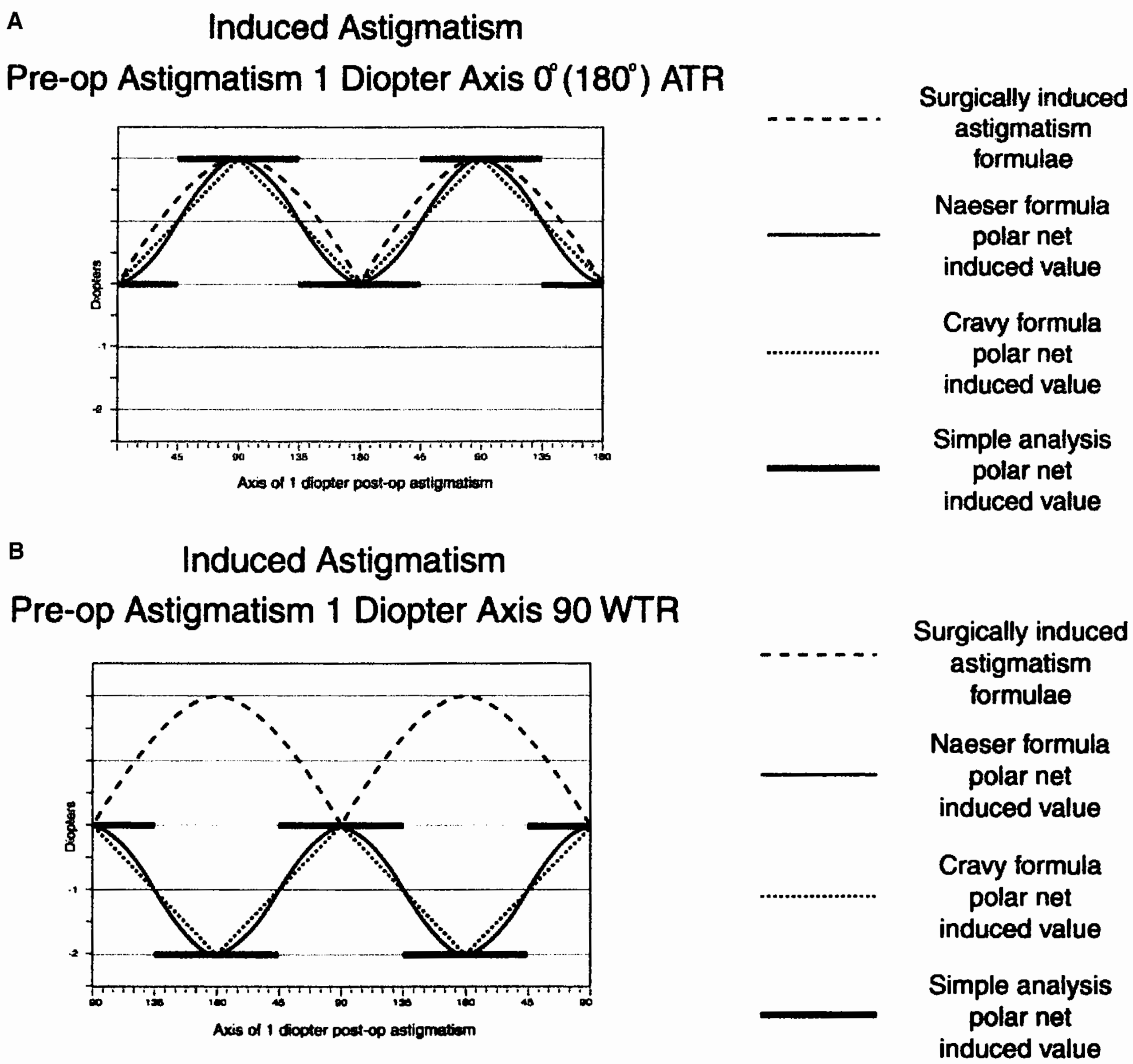
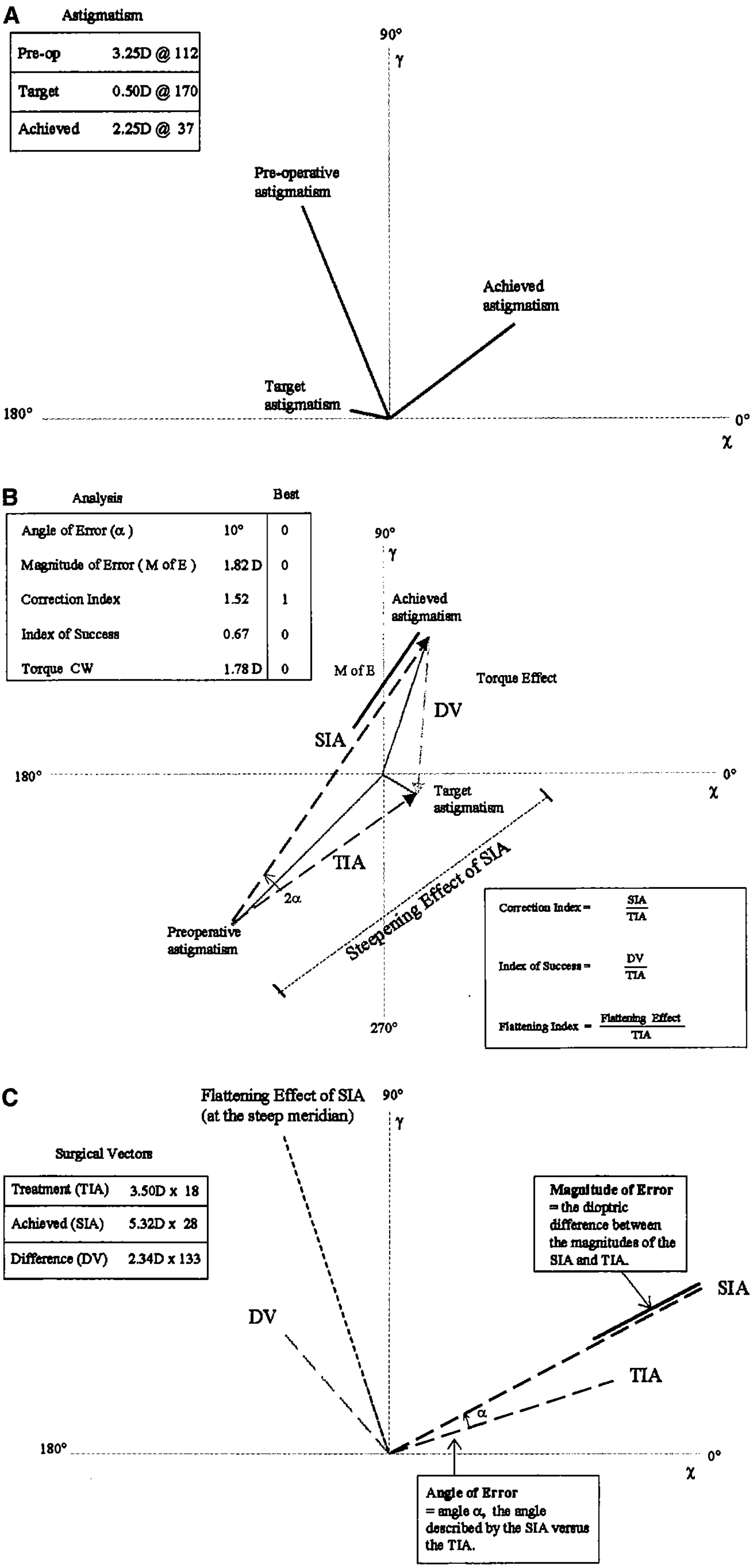


Fig. 7. *A:* The differences in behavior between various induced astigmatism calculation formulae and their cyclical nature with change according to the corneal meridian: pre-operative astigmatism being ATR. *B:* The differences in behavior between various induced astigmatism calculation formulae and their cyclical nature with change according to the corneal meridian: preoperative astigmatism being WTR. Note how Jaffe's SIA formula differentiates from the others by its constancy.

corneal steepening, possessing both magnitude (length) and direction (axis of the vector). When one is unsuccessful in hitting a ball along a chosen path into the hole, one of two events has occurred: the force with which the ball was struck was either too firm or too soft, or the direction in which it was propelled was either clockwise or counter-clockwise to that required. A combination of these two is most common. The single most comprehensive parameter of the overall success of a putt is the length required for a second putt to place the ball in the hole.

The principle underlying the use of the Alpins method in the planning and analysis of astigmatism surgery is no more complicated than this. The intended effect of the astigmatism surgery (the path from the ball to the hole), that is, the required treatment amount and its direction, is the target induced astigmatism vector (TIA).[9] The actual putt (the path the ball follows when hit) corresponds to the surgically induced astigmatism vector (SIA).[22] The difference vector (DV)[9] measures the amount and the orientation of the astigmatism treatment required to



A Astigmatism

Pre-op	3.25D @ 112
Target	0.50D @ 170
Achieved	2.25D @ 37

B Analysis

		Best
Angle of Error (α)	10°	0
Magnitude of Error (M of E)	1.82 D	0
Correction Index	1.52	1
Index of Success	0.67	0
Torque CW	1.78 D	0

Correction Index = $\frac{\text{SIA}}{\text{TIA}}$

Index of Success = $\frac{\text{DV}}{\text{TIA}}$

Flattening Index = $\frac{\text{Flattening Effect}}{\text{TIA}}$

C Surgical Vectors

Treatment (TIA)	3.50D x 18
Achieved (SIA)	5.32D x 28
Difference (DV)	2.34D x 133

Magnitude of Error = the dioptric difference between the magnitudes of the SIA and TIA.

Angle of Error = angle α, the angle described by the SIA versus the TIA.

Fig. 8. *A:* Polar astigmatism diagram: displaying pre-operative, post-operative, and target astigmatism magnitudes at their actual orientation (steep corneal meridian or power axis of negative cylinder). *B:* Double-angle vector diagram: astigmatism axis values have been doubled, but the magnitudes are unchanged. The dashed lines are vectors, with arrowheads indicating their orientation, when connecting the continuous lines displaying the astigmatisms in this double angled mathematical construct. This is the analogous golf putting green and the direction of the arrows (vectors) is also the "path of the ball." *C:* Polar surgical vector diagram: The vectors (dashed lines) are now displayed at their actual orientation by transposing them to the origin and then halving their axes derived on the DAVD. Their orientations are clearly evident and arrowheads are not necessary.

achieve the initial goal, effectively, using the golfing analogy, the second putt to get the ball in the hole after missing on the first attempt.

The DV provides a parameter of astigmatism surgery that effectively measures the error (by magnitude and axis). In practice, even though the DV represents the effect a second surgery would need to achieve in order to reach the initial goal, any further operation is best performed by freshly addressing surgical planning with the existing corneal and refractive parameters.

The various relationships between the SIA and TIA tell us whether the treatment was on-axis or off-axis, whether too much or too little treatment was applied, and the adjustments required if one had the opportunity to perform the same astigmatic correction again. The TIA quantifies the intended astigmatism treatment at the corneal plane and is the key to enabling an integrated analysis to be performed by any modality of astigmatism measurement. As already stated, all refractive astigmatism values must be converted to the corneal plane to enable a valid analysis in this way.[5–7,17,21,46]

Derivation of the vector values (magnitude and orientation) alone is not sufficient for full analysis of the surgical events. Some attempt must be made to compare the planned events with those which occurred during surgery, in numerical terms, to allow for adjustment of the next treatment or series of treatments. In this way future outcomes may be improved.

ARITHMETIC DIFFERENCES BETWEEN PRINCIPAL VECTORS: THE MAGNITUDE AND ANGLE OF ERROR

The arithmetic difference between the SIA and TIA provides us with magnitude and angle values that quantify the error value of the treatment. If the magnitude difference is positive, it represents an over-correction and if negative, an under-correction. If the angle difference is positive it represents a counter-clockwise (CCW) application of treatment, and if negative, a clockwise (CW) application of treatment to that intended.

RATIOS BETWEEN PRINCIPAL VECTOR LENGTHS

The Correction Index and Coefficient of Adjustment

The correction index (CI), which is determined by the ratio of the SIA length, representing its magnitude, to the TIA length (magnitude), is a measure of the amount of correction, and is ideally 1.0 (unity). It is greater than 1 if an over-correction has occurred and less than 1 if there has been an under-correction. The CI can be converted to a percentage of correction by multiplying the value by 100. The coefficient of adjustment (CA) is simply the inverse of the CI and quantifies the nomogram modification needed to achieve a full correction of astigmatism. Geometric means of these ratios of vector magnitudes enable nomogram adjustments of future astigmatism treatments to be made based on past outcome experience.

The measures of success obtained using this method of astigmatism analysis are both absolute and relative. The magnitude of the DV provides an *absolute* measure of the success achieved by astigmatism surgery. This vectorial comparison can be used as a scorecard for all measurement modalities (refraction, keratometry, wavefront analysis or topography).[3,10]

The Index of Success

The index of success (IOS) is determined by the relationship of the DV to the TIA, and provides a *relative* measure of surgical success. The greater the targeted change in astigmatism, the smaller the IOS value would be for any constant value of the DV, hence the more successful the surgery. Employing the golf analogy, to determine which of two putts that finished equidistant from the hole was the more successful, the answer is clearly the one that resulted from the golfer with the longer first attempted putt.

Just as one may one-putt a green and not require a second stroke, so also there may be a DV of zero. Where this is the case, the IOS value is also zero and the correction index is 1.0. The correction of astigmatism is complete on the first treatment. Here, the magnitude and angle of error are zero and the achieved astigmatism coincides with the target; no further astigmatism treatment is required.

It is important to note that a CI of 1.0 alone does not ensure complete success of the astigmatism surgery since any misalignment between the actual treatment (SIA) and the intended treatment (TIA) results in an angle of error (AE), so leaving a remaining DV and an IOS both greater than zero. In this case, the magnitude of error would be zero.

The Flattening Index (FI)

The flattening effect (FE) is the amount of astigmatism reduction achieved by the effective proportion of the SIA at the intended meridian (FE = SIA $\mathrm{Cos}2 \times \mathrm{AE}$). The FE quantifies the proportion of the SIA that is effective at reducing astigmatism (Fig. 8). A third significant ratio is the flattening index (FI)[6] derived from the FE. It is the ratio of the flattening effect of the treatment to the TIA. The flattening index is calculated by dividing the FE by the TIA and is preferably 1.0.[6]

The components of the Alpins method are shown in Table 1 and are displayed in Fig. 8. Fig. 8 displays

TABLE 1

Glossary Of Terms

Analysis of treatment

1. Target induced astigmatism vector (TIA): The astigmatic change (by magnitude and axis) the surgery was intended to induce.[9]

2. Surgically induced astigmatism vector (SIA): The amount and axis of astigmatic change the surgery actually induces.

Correction index (CI): Calculated by determining the ratio of the SIA to the TIA (what the surgery actually induces versus what the surgery was meant to induce), calculated by dividing SIA (actual effect) by TIA (target effect). The CI is preferably 1.0 (it is greater than 1.0 if an overcorrection occurs and less than 1.0 if there is an undercorrection).[7]

Errors of treatment: The arithmetic difference between the SIA and TIA magnitudes and axes.

Magnitude of error (ME): The arithmetic difference between the magnitudes of the SIA and TIA. The magnitude of error is positive for overcorrections and negative for undercorrections.

Angle of Error (AE): The angle described by the vectors of the achieved correction versus the intended correction. The AE is positive if the achieved correction is on an axis counter-clockwise (CCW) to its intended axis and negative if the achieved correction is clockwise (CW) to its intended axis.[9]

3. Difference vector (DV): The change (by magnitude and axis) that would enable the initial surgery to achieve the original target on the second attempt. The DV is an absolute measure of success and is preferably 0.[9]

Index of Success (IOS): Calculated by dividing the DV by the TIA (the intended treatment). The IOS is a relative measure of success and is preferably 0.

4. Flattening effect (FE): The amount of astigmatism reduction achieved by the effective proportion of the SIA at the intended meridian. (FE = SIA Cos2 × AE).

Flattening Index (FI): Calculated by dividing the FE by the TIA and is preferably 1.0.[6]

5. Torque: The amount of astigmatic change induced by the SIA that has been ineffective in reducing astigmatism at the intended meridian but has caused rotation and a small increase in the existing astigmatism. Torque lies 45° CCW to the SIA if positive and 45° CW to the SIA if negative.[6]

6. Nomogram calculator for astigmatism: An additional parameter is available from This method of astigmatism analysis that enables the achievement of a full correction of astigmatism magnitude in future treatments based on past experience. This is:

Coefficient of Adjustment (CA): (derived by dividing TIA by SIA, the CI inverse), to adjust future astigmastism treatment (TIA) magnitude. Its value is preferably 1.0.[9]

Analysis of Ocular Status

1. Ocular Residual Astigmatism (ORA): vectorial measure of the non-corneal component of total refractive astigmatism, that is, the vector difference between refractive and corneal astigmastism calculated in diopters.[7]

2. Topographic Disparity (TD): a vectorial measure of irregular astigmatism calculated in diopters.[5]

Analogous Parameters for Parallel Comparisons of Spherical Change at Corneal Plane[3,10]

1. Spherical correction index (S.CI):

$$\frac{\text{Spherical equivalent correction achieved}}{\text{Spherical equivalent correction targeted}}$$

2. Spherical Difference (SDiff):
[Spherical equivalent achieved − spherical equivalent targeted] (absolute)

3. Index of success of spherical change (S.IOS):

$$\frac{\text{Spherical difference}}{\text{Spherical equivalent correction targeted}}$$

4. To express indices as percentages:
Percentage of astigmatism corrected: CI × 100
Percentage of astigmatism reduction at the intended axis: FI × 100
Percentage success of astigmatism surgery: (1.0 − IOS) × 100
Percentage of sphere corrected: SCI × 100
Percentage success of spherical surgery: (1.0 − Sph IOS) × 100

corneal values for astigmatism and vectors, both on a DAVD. Also displayed in Table 1 are all the astigmatism values, surgical vector values, and an analysis of errors and correction.

The DAVD is constructed by doubling all meridian values of astigmatism and displaying these values (astigmatisms) at their respective double angled location. The vectors are determined by joining the heads of the astigmatism displays with dashed lines. To view the surgical vectors at their actual position, their axis value is halved and the tail of the vector is located at the origin of the polar surgical vector graph.

ADVANCED VECTOR ANALYSIS TECHNIQUES

The Alpins method is an acknowledged method[25–28] for the analysis of astigmatic outcome and has been employed in many published studies since first used by the authors of this article.[15,42] It employs all advanced vector analysis techniques that are useful in analysis of samples and populations.

Averaging Vector Magnitudes

Vector magnitudes can be averaged without reference to their respective axes to determine their arithmetic *magnitude mean* for the group. For example, a magnitude mean of DVs for a series of procedures usefully compares success in absolute terms when examining the effectiveness of astigmatism surgery techniques.[3] It is not useful to average axes of astigmatism or vector values. This is true when the values are presented in their semi-meridian range (0–180°)[20] or where the axes are doubled. A group of vector axes can be usefully averaged only when in association with their respective magnitudes determined in a head-to-tail summation.

The statistical examination of means of vector magnitudes alone, without reference to their respective orientations, is a valuable and valid technique in astigmatism surgery analysis. It has been asserted in the past that obtaining a mean of vector values is not amenable to statistical procedures[2,32] and hypothesis testing, because the value obtained does not readily allow the determination of a variance of the data.[24] The mean vector magnitude is a valid summary of the central tendency of a group of vector magnitudes, and the formula for variance applies when a mean is calculable.

$$\sigma^2 = \Sigma(X - \mu)^2/n - 1$$

The variance is the squared sum of all the deviations from the mean, divided by the degrees of freedom. Hypothesis testing is not dependent on the calculation of variance. When the data is skewed and the mean is not a valid summary of central tendency, then neither is the variance a valid measure of dispersion, so that nonparametric statistics can then be used[3] to test hypotheses.

The magnitude of the vector can be summated with regard to each vector's orientation as discussed above to determine a summated vectorial mean of the group. This summated vectorial mean is always less than the mean vector magnitude, and the greater the difference between the two (or the smaller the ratio of the vector to arithmetic mean) the less any overall trend is evident.[3] In the display of aggregate analysis, it is helpful to show the individual vectors and the summated vector mean in the same polar diagram as in Fig. 9A.

Vector Summation

Vector magnitudes can be averaged with reference to each of their respective axes if vector summation is performed (Fig. 9B). A *summated vectorial mean* (also termed "centroid"[21]) of the DVs, for example, can be obtained for any group of surgeries by summing all of the individual vectors at their own orientations head-to-tail, and then dividing the net resultant vector's total length by the number of its individual components.[30] This determines a magnitude and axis for the summated vectorial mean to enable a trend analysis of aggregate data. This magnitude will be less than the arithmetic magnitude mean of DVs described above, and is useful to determine if any net systematic errors are occurring with any group of patients' aggregate data. The greater the difference between magnitude and summated vector means, or the closer the summated vector mean is to zero, then the more likely the magnitude mean of the DVs is due to random rather than systematic error.[3] The comparison of vector mean to arithmetic mean of the DVs can also be expressed as a ratio.[3]

The vectorial mean of any group of vectors usefully compares astigmatism surgery techniques and identifies trends occurring in multiple treatments. This value is preferentially displayed on a polar (0–180°) diagram after being derived by vector summation on a double angle vector diagram. This mode of display is simpler and more clinically intuitive, as a vector, such as the SIA, displayed in this way indicates the actual meridian of the eye where the maximum ablation effect occurred.[3] Holladay et al., who have described this value obtained as the "centroid" value,[21] recommend its display at double its actual axis, which, as a consequence, renders it difficult to relate the changes to the eye. A number of authors have advised against this approach as it also would appear to introduce unnecessary complexity into an already demanding subject.[9,16,33] This complexity of double angle display of vectors[39] would potentially cause even greater confusion when it is necessary to display concurrently the lower half (180–360°) of the cornea where asymmetrical treatment and analysis is performed for irregular astigmatism. By dealing with vectors in the polar mode, the process is simplified and this confusion is avoided.

Averaging Vector Angular Separation

The angular separation between any two vectors, such as the SIA and TIA, can be arithmetically averaged to determine a mean angle of error value for a group of procedures, as this parameter is not a vector value. This can be performed by either recognizing positive and negative values to achieve the mean arithmetic net error, or by using absolute values to determine the average overall error in the application of astigmatism treatment. This information provides an adjunct to the standard deviation values of the mean arithmetic error. Scatter plots with a range of −90° to +90° are valuable in displaying multiple individual values of the angle of error.[3]

Resolving Vectors

Resolution of vectors is employed for calculating flattening and steepening effect that describes the attempted changes occurring in all cataract and

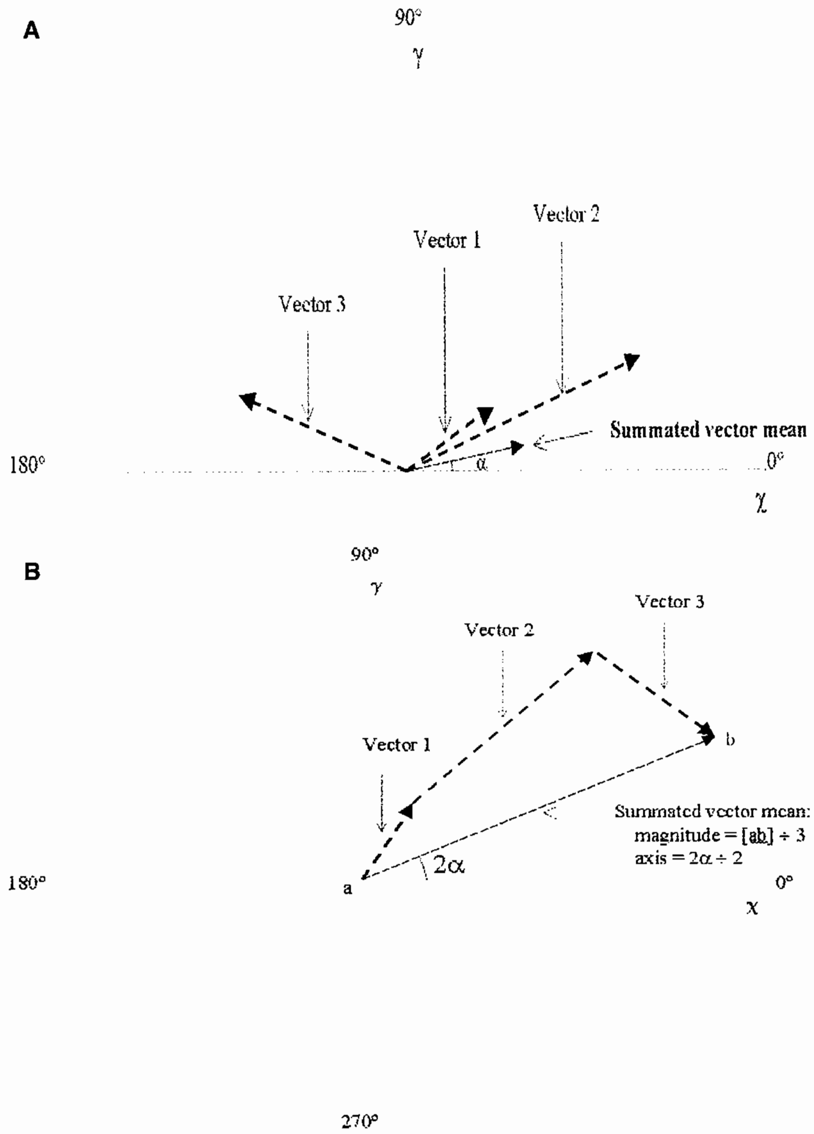


Fig. 9. A: Polar analysis of vectors. Multiple vectors are present on a polar diagram at their actual orientation, as they would appear on an eye. When averaged by addressing both magnitude and their axes, some of their effects cancel each other out. The summated vector mean resultant, determined in Fig. 9B, is shown in red at its orientation. *B:* The vectors displayed on the polar diagram have their axes doubled to be displayed on a DAVD, and then are transposed in a head-to-tail summated fashion. The red dashed line connects the first tail at the graph origin to the last vector head. To determine the magnitude of the summated vector mean, the total length is divided by the number of component vectors and its axis is then halved to return to a polar diagram display as in Fig. 9A.

refractive surgical techniques.[6] It performs the tasks intended by Naeser's modified formula and Holladay's calculations, simplifies the terminology, and more readily achieves the correct angular component.[14]

$$K_{\text{Flattening/Steepening}} = \text{SIA cosine } 2\alpha$$

Where α is the angular amount of treatment axis misalignment (angle of error). This formula enables calculation of the changes induced by the refractive cataract surgery incision (or the laser ablation) at the intended axis of cylinder reduction. It simplifies the calculation of the effect of the treatment by employing only the values of the SIA and the angle of error that quantifies the angular misalignment of the treatment, whether by incision or ablation. The loss of effect of astigmatism reduction caused by off-axis treatments has, in the past, been overstated.[1]

The astigmatic change at any chosen orientation[6] can be determined by resolving the SIA (on a DAVD) to examine how much of its effect is occurring at that chosen reference axis. When performing surgery to reduce astigmatism, the relevant axes to examine, by resolving the SIA vector for each, are the following:

- At the pre-operative astigmatism (or treatment) meridian to determine how much flattening effect of the SIA has occurred effectively to reduce astigmatism (Fig. 8)
- At 45° (or 90° on the DAVD) to the pre-operative astigmatism (or treatment) meridian to determine the proportion of the SIA change that has occurred that does not assist in reducing astigmatism but instead has produced rotation and increase of the existing astigmatism by *torque effect.* This torque is clockwise if a negative value is obtained, or counter-clockwise if positive.

The same principles as flattening and steepening apply to *against-the-wound* and *with-the-wound* change,[20] using the wound site as the reference meridian. The value for the change in polar directions with-the-rule (WTR) and against-the-rule (ATR), as is achieved using the Naeser[34] and Cravy[12] formulas, is derived by projecting the SIA onto the 180°-meridian on a DAVD as the reference axis, so determining the polar change at 90°.[6] Naeser has also subsequently described its projection onto other meridia.[35,36] In the past, various non-vector approximation formulae[11,31,40] have been used to differentiate WTR, oblique, and ATR astigmatism.

The key essential features of the Alpins method of vector analysis are displayed in Fig. 8. This simplified view may assist authors in achieving a clearer understanding of the principles of astigmatic analysis[2,32] to avoid inaccuracies in the presentation and interpretation of published results.[8,29] The complexities of sign notation with steepening/flattening, WTR/ATR, over- and under-correction, torque and off-axis treatments, whether they be CW or CCW, can be challenging to those not familiar with the trigonometric conventions that prevail.

Astigmatic Outcomes Analysis—What Is Required?

COMPREHENSIVE UNDERSTANDING

In general terms, there are three indices that enable examination of the relationship of three separate vectors to the treatment vector (the TIA) and comprise a complete approach to astigmatism analysis:

1. The index of success (IOS), a measure of relative success derived from the DV
2. The flattening index (FI), calculated from the flattening effect (FE) achieved by the effective proportion of the SIA at the axis of the TIA
3. The correction index (CI), which is the overall astigmatism correction achieved by the SIA

When examined together, the three provide a comprehensive understanding of any astigmatic change and of the proportion of the astigmatism treatment that has been effectively applied.

INTERPRETATION OF RESULTS

Using the above method, the CI shows the surgeon how efficiently the laser is correcting astigmatism. The IOS shows how the astigmatic results of this laser compare with other lasers and other techniques. The FI shows how effective the treatment was in reducing astigmatism. Favorable changes at the preoperative astigmatism meridian are quantified by flattening effect and ineffective changes are evaluated by torque. These values can be displayed graphically at the treatment axis on a polar diagram.[3] This method provides a comprehensive understanding of induced astigmatic change and offers significant advantages by enabling an integrated examination of all changes applicable to keratometry, topography, manifest refraction, or wavefront analysis values.

All the parameters included in Table 1 are useful variables when examining astigmatism as part of one of two fundamental groupings of data, corneal or refractive. When performing a study, a choice can be made to select the particular parameter(s) to compare variables of interest in any one or a group of eyes. Some variables, such as angle of error or torque, can produce a positive or negative value that is cyclically direction specific. Where means are required, they can be either arithmetic and sign sensitive to examine trends, or absolute, assigning absolute values to negative signs to examine total spread of errors.

Spherical powers are a third axis perpendicular to the plane of the astigmatism[3,10,18,44] and together the analogous spherical and either of the astigmatic analyses, refractive or corneal, can define three dimensional dioptric power spaces either in corneal or refractive terms. This incorporates the error of sphere in addition to the astigmatic error. Similarly, the error can be described in three-dimensional terms if desired, with a combined parameter of the difference vector and the spherical difference.[3]

Integrated Analysis

The adjustments necessary due to the errors that are gleaned from a complete 360° corneal analysis can be performed using refractive or corneal data (for example, by examining upper and lower halves of the cornea separately in cases of irregular astigmatism). By examining both sets of parallel data, suitable nomogram adjustments can be fed back into a laser algorithm at their various sectorial orientations to correct and refine treatments over time to an increasingly accurate end point. In this way, the two diagnostic modalities of corneal and refractive values can be combined with advanced planning and analysis techniques to provide a single integrated module to treat refractive errors associated with regular and irregular astigmatism.[4]

Addition of the sphere equivalent of a spherocylinder yields a third dioptric variable on a third axis orthogonal to the x–y astigmatic plane (a third dimension), and a vector can be drawn from the coordinate origin of this three-dimensional space to the point described by the three coordinates. Such a vector expresses the overall blurring effect of that spherocylinder.[37,43,44] By extension, a target induced spherocylindrical vector could be derived from preoperative data and the desired remaining postoperative error (usually emmetropia). This could then be compared with a surgically induced spherocylinder vector derived from the pre- and postoperative data, and similar indices of success could be derived as for pure astigmatism. This might be a useful measure of the success of overall refractive change, particularly for group data, but it is thought be of limited value in nomogram development since it summarizes the effect of the several components of a refractive treatment (sphere, cylinder, and axis).

It should be noted that the three-dimensional vector describing the overall blur of a spherocylinder is not, however, sensitive to the sign of the error. Furthermore, as with all methods based on polar value, there is an assumption of orthogonality of the steep and flat axes which may not be appropriate in all types of analyses, especially those based on videokeratoscopy where these axes are seldom thus aligned. It may only be possible adequately to analyse this kind of optical system by employing a shift to thick lens optics and a full description of the optics of both the eye and the surgery undertaken to alter it.[18,19] Much of the debate about the proper analysis of astigmatism is due to the application of the oversimplifications and assumptions of thin lens systems to a thick lens system, such as the eye.

Summary

Vector analysis is highly useful to the surgeon in that it not only provides the basis for uniformity in comparative analysis of surgical results, but it also enables assessment of operative technique by determining success and identifying errors and the essential nomogram adjustments required to improve performance to achieve improved surgical outcomes.

Method of Literature Search

MEDLINE and PubMed were searched to March 2003 using the search words *astigmatism, keratometric astigmatism, refractive astigmatism, vector, simple analysis, surgically induced astigmatism, target induced astigmatism, difference vector, with-the-wound, against-the-wound, astigmatism change, axis shift.* Relevant references from articles thus obtained were then accessed. Standard texts were included and references there contained were accessed where relevant. The original literature developing this technique is relatively small and appears mostly in English. Non-English language literature was searched where English language abstracts were available.

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