Raytracing for IOL Power Calculation

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Raytracing is defined as a calculation method for single rays passing through an optical system. Starting at a given point and at a given angle relative to the optical system (e.g. to the optical axis), the ray is refracted on each optical surface according to Snell’s law, thereby each time changing its direction. Unfortunately, the final direction of a ray which has passed through an optical system with more than one refracting surface cannot be calculated in closed formulae. Equations with nested sine expressions like Snell’s law are called transcendental equations. They are principally not solvable in closed formulae but need an iterative, numerical solution.

Nearly all state-of-the-art consumer optical devices are calculated with the help of raytracing software, not with Gaussian optics. This could suggest that raytracing is a modern approach. But in fact, the principles of raytracing have been developed in the early 17th century, by scientists like Willebrord Snellius, Christian Huygens, René Descartes and Pierre de Fermat. But the calculation of optical systems with raytracing is not feasible without the aid of a computer. Therefore, even if all physical principles were understood in the 17th century, optical devices could not be calculated. It needed about 150 years until Karl Friedrich Gauss developed a calculation method utilizing Snell’s law in a simplified version. He approximated the sine by the first element of series expansion of the sine, i.e. the angle (arc) itself. With this approach, optical systems with more than one refracting surface can be calculated in closed formulae such as the well-known IOL formulae.

Gaussian optics is only valid in optical systems centered relative to an optical axis. In addition, the rays of interest must have only very small angles relative to the optical axis. Gaussian optics cannot describe phenomena like spherical aberration which causes a focus shift with changing pupil size, in ophthalmology known as “night myopia”.

The currently used 3rd-generation IOL calculation formulae do not even utilize the full capabilities of Gaussian optics. Instead, they apply two ad-
ditional simplifications: anterior and posterior corneal surface are combined to one surface, and the IOL is approximated as a “thin lens”.

The combination of anterior and posterior corneal surface to one surface needs assumptions about the ratio of anterior and posterior curvature radii. For a given ratio a fictitious “corneal refractive index” is calculated which is smaller than the refractive index of the corneal material and smaller than the refractive index of aqueous humor, because the posterior cornea acts as a minus lens. Differences in this fictitious corneal refractive index is one of the differences between the IOL formulae.

The second difference between the formulae is the assumed position of the IOL. If the so-called “formula constants” are adjusted correctly, this position is the position of a thin lens of the appropriate power which gives the eye the required target refraction. This fictitious position is often called “effective lens position” (ELP). As a consequence of different corneal refractive indices for the different formulae, the ELPs must differ too. Thus, the different formulae in fact describe different physical systems.

Raytracing applied to IOL calculation utilizes a description of the pseudophakic eye in which anterior and posterior corneal surface ideally should be topographically measured. The IOL is described by anterior and posterior central curvature radius, asphericity (if any) of the surfaces, central thickness and index of refraction. The position of this IOL is the true geometrical position, e.g. defined by the anterior chamber depth (ACD), the distance between posterior corneal and anterior IOL apex.

The postoperative IOL position cannot be determined preoperatively. With respect to the prediction accuracy of postoperative IOL position, there is no principal advantage of raytracing compared to calculations in Gaussian optics. But any prediction method for postoperative ACD used in raytracing can (and should) be directly compared to corresponding ACD measurements. With the fictitious ELP of the IOL formulae such a direct comparison is not possible.

ACD measurements can be performed by partial coherence interferometry (PCI) with high accuracy. On the other hand, ACD or ELP could also be back-calculated from the postoperative refraction. But such a backcalculation is much less accurate than a PCI measurement because it contains error contributions from subjective or objective refraction and from IOL manufacturing (mislabelling error). The ACD prediction method used in the raytracing therefore should be based on and be consistent with PCI measurements of postoperative ACD.

The measured postoperative ACD can principally depend on several parameters such as time after surgery, rhexis size, IOL haptic angulation and thickness, and IOL material. Measurements of these influencing parameters therefore are essential for the quality of a postoperative ACD prediction method.

It can be assumed that size and position of the crystalline lens have a significant predictive impact on postoperative ACD. But sonographic lens thickness measurements have high measurement errors, and optical mea-
surements are mostly not available with current equipment. Preoperative ACD alone, i.e. without lens thickness, disguises the influence of the growth of the crystalline lens with age.

Good correlations can be found between postoperative ACD and axial eye length or corneal radius. But it should be kept in mind that all said parameters are also highly correlated with each other. Therefore, it may be dangerous (and mathematically incorrect) to use them as independent predicting variables in a multiple regression fit approach.

The simplest ACD prediction method in our raytracing uses only a nearly linear relationship between axial eye length and postoperative IOL center distance to posterior corneal apex. The postoperative ACD can be calculated by subtracting half of the IOL thickness. The accuracy of this ACD prediction algorithm can be improved with the additional information of the crystalline lens position and thickness which is meanwhile available also from optical measurements providing a higher accuracy than ultrasound.

Despite the mentioned principal differences between raytracing and 3rd-generation IOL formulae the prediction errors between both approaches do not differ significantly for mean-sized eyes. In these eyes, the uncertainty of postoperative IOL position is a more important source of error than the mentioned systematic differences between the optical systems which are described by the formulae. This, however, changes for eyes far away from the mean-sized normal eye. The theoretical differences shown in fig.1 are confirmed by corresponding clinical data particularly of very short and very long eyes.

Also eyes after corneal refractive surgery differ from normal eyes. In these eyes the highest errors occur from keratometrically measured corneal radii. The reason is that keratometers are adjusted to spherical or moderately prolate aspherical corneae. But eyes after refractive surgery often have oblate aspherical cornea. The resulting errors can be avoided by replacing keratometry by topography, combined with an adequate algorithm included in the raytracing that extracts corneal vertex radii together with corneal asphericity in the optical zone.

A second source of error in eyes after refractive surgery is the abovementioned fictitious “corneal refractive index” which assumes a constant ratio of anterior to posterior corneal radii. But when anterior radii are modified in refractive surgery, the said ratio must change too. The solution of the problem is an additional measurement of posterior corneal curvature, e.g. by Scheimpflug or OCT techniques, see fig.6.

If topographic measurements of anterior and posterior cornea are performed in all eyes, there is no need to distinguish between normal eyes and eyes after refractive surgery.
Figure 1: **Refraction Prediction Differences**

The theoretical differences between OKULIX raytracing and the Haigis, SRK/T, Hoffer-Q and Holladay formula are shown as functions of axial eye length in the range between 16mm and 36mm. Such results also depend on corneal radius the influence of which in clinical data causes a scattered diagram in which systematic differences are often not clearly recognizable. To avoid this, a linear relationship found in a clinical sample has been applied here to simplify the dependence of the corneal radius $R$ from axial eye length $a$: $R = 0.09521a + 5.4016$

Raytracing is used as reference. Its prediction difference therefore set to zero.

Extreme axial eye lengths are very rare. The grey histogram (ordinate on the right) shows the axial length distribution of 2888 consecutively operated eyes (Data P. Hoffmann, Castrop-Rauxel). Note that all curves are cutting each other at the location of the distribution frequency peak.
Figure 2: **Differences in Assumed IOL Position**

The IOL positions for OKULIX raytracing and for the Haigis, SRK/T, Hoffer-Q and Holladay formula are shown as functions of axial eye length. The same relation between corneal radius and axial eye length as in fig. 1 is assumed. For the formulae, the ELP (explanation see text) is shown, for raytracing the IOL center position (solid blue) and the ACD (dotted blue).
Figure 3: **Prediction Differences**

The prediction errors of OKULIX raytracing and of the Haigis, SRK/T, Hoffer-Q and Holladay formula are shown as functions of axial eye length. The data are fitted by 8th-order polynomials. The distribution histogram is shown in grey, with scale on the right (Data P. Hoffmann, Castrop-Rauxel).
Figure 4: **Mean Absolute Errors**

The mean absolute errors of OKULIX raytracing and of the Haigis, SRK/T, Hoffer-Q and Holladay formula are shown as functions of axial eye length. The data are fitted by 8th-order polynomials. The distribution histogram is shown in grey, with scale on the right (data P. Hoffmann, Castrop-Rauxel).
Figure 5: Prediction Differences in Very Long Eyes
The prediction errors of OKULIX raytracing and of the Haigis, SRK/T, Hoffer-Q and Holladay formula are shown as functions of axial eye length. The distribution histogram is shown in grey, with scale on the right (Data P. Hoffmann, Castrop-Rauxel, K. Petermeier, Tübingen).
measured Lasik correction (spher. equiv.)

Figure 6: Eyes after Lasik

IOL calculation prior to and after Lasik was performed with OKULIX ray-tracing in the same 17 eyes. In both cases, anterior corneal topography and spatially resolved corneal thickness (TMS5, Tomey) was measured, together with IOLMaster (Zeiss) axial eye lengths. Refractive prediction was calculated with the same IOL (AMO, Ar40e) producing a refraction close to emmetropia for each eye under the pre-Lasik condition. The pre-post-Lasik differences of these refractions from IOL calculation should theoretically be identical to the Lasik correction. A logically equivalent procedure can be applied to the calculation methods proposed by Rosa et al [10] and by Haigis [2] as well as for the standard Haigis- and SRK/T-formula. Note that these calculations are independent on the so-called “formula constants” because their impact disappears with the calculation of differences.

(Data T. Hofmann, Binningen, Switzerland).
References


